average), and contained 41·8 grs. of nitrogen daily. It was largest on the Sunday, and the fæces then contained an increase of nitrogen equal to the quantity lost by the urine on that day. Hence the assimilation was defective, but it was increased by labour.

The author then showed the relations of urea and carbonic acid, and their dependence upon food, season, and period of the day, and discussed the relation of urea to exertion and nutrition, showing that unless there be continued waste of the nitrogenous tissue, there cannot be any important increase of urea from exertion.

The paper was accompanied by numerous explanatory tables and curves.

The following communication was read :—

"On the Theory of the Polyedra." By the Rev. T. P. Kirkman, M.A., F.R.S., and Hon. Member of the Literary and Philosophical Society of Manchester. Received May 10, 1861.

(Abstract.)

The complete resolution of the problem of the polyedra embraces the construction of all P-edra Q-acra, with an account of the symmetry of the solids when symmetrical. Such construction being impracticable from the magnitude of the task, it is desirable that a method should be found of classifying and enumerating the P-edra Q-acra, so that from this knowledge of the inferior polyedra, the same can be obtained concerning the higher, without any constructions, and without any tentative process.

I have found that all attempts to enumerate a separate and well-defined family of the P-edra Q-acra, beyond that I have called 'The partitions of the R-pyramid' (Phil. Trans. 1858), have been fruitless, and that the simplest method of solving the problem is to solve it entirely.

It is necessary first to have an accurate classification of polyedra as to symmetry. This is—

1. Zoned symmetry;
2. Zoneless axial symmetry;
3. Mixed symmetry, both 1 and 2;
4. Neuter symmetry, neither 1 nor 2;
5. Asymmetrical.

1. *Zoned symmetry.*—A polyedron may have zones of one, of two, or of three configurations. A zone divides the solid, when it is constructed with the greatest possible symmetry, into halves, of which one is the reflected image of the other. Zoned polyedra are,—

1. Monozones;
2. *m*-zoned monaxines;
3. Zoned triaxines;
4. Zoned monarchaxines, having secondary axes;
5. Zoned polyarchaxines, which have the axial systems of the regular polyedra.

The intersection of two zones is a *zoned axis.*

The *zonal signature* gives an account of the number of zonal summit faces and edges, whether polar or non-polar; but not of the number of edges in the zoned features.

2. *Zoneless symmetry.*—This is *m*-ple *repetition* of configuration, in revolution about a zoneless axis. *An axis is janal* if two opposite eyes can read at the poles configurations C C′, which are either identical, or one the reflected image of the other; otherwise the axis is *heteroid.* A zoned janal axis is *objanal* when C is C′ turned through two right angles. A zoneless janal axis is *contrajanal* when C is the reflected image of C′. The polyedra of this symmetry are,—

1. Zoneless heteroid monaxines;
2. Zoneless monaxine contrajanals;
3. Zoneless triaxines;
4. Zoneless monarchaxine janals, having secondary axes;
5. Zoneless polyarchaxines, having the axial systems of the regular polyedra.

3. *Mixed symmetry.*—In this there is both a zone or zones and one or more zoneless axes. The solids are,—

1. Monozone monaxines;
2. *m*-zoned homozones, having *m* zoneless axes.

A zoned janal axis is *homozone* when the solid has zones of one configuration only; otherwise the janal zoned axis is *heterozone,* i. e. the solid has zones of more than one configuration.

4. *Neuter symmetry.*—There is neither zone nor zoneless axis, but the edges of the solid fall into pairs *ab*, *a′b′* diametrically opposite, of which edges one is the reflected image of the other.

The polyedra of this symmetry are *contrajanal anaxine polyedra.*

5. The *asymmetric polyedra* are the most numerous of all.

The data required for the enumeration of the P-edra Q-acra are the following:—

A. A table of the hexarchaxine, tetrarchaxine, and triarchaxine P-edra Q-acra and Q-edra P-acra, both zoned and zoneless, with all the zonal signatures, and an account of the terminations of their principal axes, and with the *zonoid signatures*.

The *zonoid signature* in a zoneless or mixed symmetry gives the number of zoneless polar faces, summits, and edges.

B. A table of *janal poles*, containing an exact account of every polar feature terminating a janal axis, with the repetition if it be zoneless, and with the traces and zones if zoned.

C. A table of poles, containing an exact account of every polar feature terminating either a janal or a heteroid axis, with zonal signatures and traces when they exist, or with the zoneless repetition.

D. A table of faces and summits having a diagonal trace of a single zone, with the zonal signature.

E. A table of objanal monozone faces and summits, with their zonal and zonoid signatures.

F. A table of contrajanal anaxine pairs of edges, with the faces A B intersecting in them.

G. A table of all edges (A B) with their intersecting faces, the polar edges with their zones, the monozone zonal and epizonal edges each with its zone. An *epizonal edge* is cut by a zone.

These data being first obtained for the P-edra Q-acra and for the Q-edra P-acra, the numbers of the following solids and faces and summits are exactly known for both P-edra Q-acra and Q-edra P-acra; and we suppose that all the inferior polyedra are registered in like manner:—

a. The *m*-zoned monarchaxines with their zonal signatures, and their principal poles;

b. The zoned triaxines with their zonal signatures;

c. The *m*-zoned homozones with their zonal and zonoid signatures, and their exact principal poles;

d. The zoneless monarchaxine janals with their repetition and zonoid signatures, and their principal poles;

e. The zoneless triaxines;

f. The monozone monaxines with their zonal signatures and their poles;

g. The zoneless monaxine contrajanals with their repetition and their poles;

h. The *m*-zoned monaxines with their zonal signatures and poles;

i. The monozones, with their zonal signatures;

j. The zoneless monaxine heteroids with their axes and repetition;

k. The contrajanal anaxine polyedra;

l. The asymmetric polyedra;

m. The monozone A-gons and A-aces with their traces and zonal signatures;

n. The objanal monozone A-gons and A-aces with their zonal and zonoid signatures;

o. The janal anaxine A-gons and A-aces;

p. The asymmetric A-gons and A-aces.

And these numbers being registered along with the data preceding for all signatures, we have a complete classification and enumeration of the P-edra Q-acra and Q-edra P-acra, which can be continued to any values of P and Q.

The difficulty lies in the obtaining of the data A B C D E F G. We begin with (C, D, and G).

Analysis of a polar or monozone summit of a P-*edron* Q *acron* V.

Let p' be a polar or monozone p-ace, which, if polar, is either a zoned or zoneless termination of an axis. If $\frac{r}{2}$-zoned or of r-ple zoneless repetition, there is a sequence of configuration r times read about the pole.

Any summit m through which lies the triangular section $p'mn$ of the solid is a *deltotomous summit* about p'; and if mn be an edge, it is a *deltotomous edge*, about p', or of p'.

If the deltotomous edge mn be in two faces ff', of which f has no deltotomous summits about p' except mn, while f' either has more than two deltotomous summits about p', or is a triangle mnr having r collateral with p', mn is *a primary effaceable of* p'.

If the deltotomous edge mn be in two faces ff', of which neither has any deltotomous summits about p' besides mn, nor is a triangle mnr having r collateral with p', mn is a *secondary effaceable* of p'.

The summit p' has e ($\gtreqless 0$) effaced effaceables. These can be restored in one way only, since each (mn) must complete a face f which has no deltotomous summit besides mn. Let them be restored. We have the *perfect summit* p' (*i. e.* a summit which has no effaced effaceables) of a ($P+e$)-edron Q-acron $V_{,}$, which has $E \gtreqless e$ effaceables.

The process whereby the data C, D are obtained is the construction in groups of all *perfect* polar p-aces and monozone p-aces having a diagonal zonal trace, of all the ($P+e$)-edra Q-acra ($V_{,}$), which can be reduced by effacement of e effaceables to a polar or monozone p-ace of a P-edron Q-acron V. These polyedra $V_{,}$ can be all constructed about their perfect poles and registered; for no result of effacement is small enough to be employed in the construction of a polyedron ($V_{,}$). Each perfect p-ace p' constructed has $E(\gtreqless e)$ effaceables, which are registered with the signatures of symmetry; so that all results of effacing $e(\gtreqless 0)$ of the E effaceables about any registered perfect p-ace are exactly known without repetition, and without enumeration of two summits, of which one is the reflected image of the other.

The process of this construction of perfect p-aces p' is the converse of *the reduction of* p'.

In *the reduction of* p', we remove all the rays of p', whereby we lay bare either a polyedron or a *reticulation*, which has among its linear sections the E effaceables of p'.

If the effaceables of the reticulation are all secondary, it is a *full reticulation*, which is an agglutination of polyedra cohering by those secondary effaceables, which are linear sections, and the only linear sections, of the reticulation.

If the reticulation has no effaceables, it is a *plane reticulation*, being simply a polygon partitioned by certain diagonals.

If there be one or more primary effaceables, we have a *mixed reticulation.*

The mixed reticulation always reduces in one way only by sections in its *external primary effaceables*, to a subject reticulation, which is either mixed or full or plane, or else to a polyedron. And, by continuing this reduction, we always obtain finally a full reticulation, a plane reticulation, or a polyedron, $\frac{ri}{2}$-zoned, or of ri-ple zoneless repetition.

The primary effaceables of a mixed reticulation are the joints or

seams by which it can be either taken to pieces or constructed, in one way only. And the portions into which it breaks up by these joints, are either polyedra, or full reticulations, or plane reticulations, of which we have an account in our register.

We *first suppose given* all plane and full reticulations, as well as the inferior polyedra, with all their signatures. By means of these, imposed as marginal charges on a subject reticulation, we can construct and register exactly all polar and monozone mixed reticulations, with their repetition and zonal signatures, and with their effaceables, which are always edges that have been loaded with marginal charges.

Every reticulation, plane, full, or mixed, is registered with its *marginal signature.*

The marginal signature exhibits to the eye all that is requisite to be known either for the *coronation* of the reticulation by a p-ace p', whereby it becomes a polyedron, or for the construction on it of higher reticulations. The marginal signature to be constructed can always be exactly written; and there is a given number of ways of constructing it on the subject signature, which can be registered, and the number is obtained by inspection of our signatures. We always see in a mixed marginal signature what is removeable by sections in the external primary effaceables, and in a full reticulation we see what is removeable by external secondary effaceables.

The rules for charging a subject marginal signature, so as to construct another signature upon it, are :—

1. No primary effaceable which is external in the subject is to be external in the constructed; therefore the subject *compartment* standing on every external primary effaceable must receive at least one charge, which charge will be a *compartment* of the new signature.

2. Solid charges (polyedra or full reticulations) are imposed on plane marginal triangles of the subject. Plane reticulations, which give plane compartments, are imposed on solid marginal edges of the subject by a marginal triangle of the charge, which is lost in the operation, or supposed to be cut away. Thus a marginal triangle is lost by every charge imposed, whether plane or solid.

We handle all plane reticulations only by their marginal triangles, given by their signatures, and thus we escape the necessity of keeping any account of their summits.

3. A plane submargin of the subject is never charged.

4. Every charge adds a primary effaceable in a mixed reticulation, and may add any number of secondary effaceables, if a solid charge.

5. If the penesolid 402, which consists only of two marginal triangles, is employed to form a compartment of the new signature (which is the compartment $(2)^r$, or (2) r times repeated in the r-ple marginal signature), the resulting reticulation is *closed* to further marginal increments, and can only be crowned.

The zonal and zonoid signatures of reticulations give an account of all zoned and polar effaceables, so that after coronation of a reticulation we can always enumerate the results of effacement, with their symmetry and signatures. Any effaceable can be effaced independently of others.

Asymmetric mixed reticulations are never constructed, unless such as can be crowned by a line; we thus escape the construction of the most numerous class of mixed reticulations. And asymmetric coronations, or asymmetric results of effacement, are never registered, unless the coronation be by a line, or the effacements be about a *perfect edge*.

An edge is a 2-ace summit which has always $e(\gtreqless 0)$ effaced effaceables that can be restored in one way only. When they are restored we have a perfect 2-ace, which is to be constructed and registered, whether symmetrical or not.

The removal of an edge lays bare in all cases either a polyedron or a *penesolid*, which may be either plane, full, or mixed.

A *penesolid* is a reticulation which can be crowned with a line, thus becoming a polyedron.

The order of the marginal features is not regarded in the marginal signature of a reticulation, as we keep no exact account of the faces about the crowning p-ace; but the marginal signature of a penesolid is always so written that the exact faces of the crowning line shall be known by inspection of the signature. A penesolid may have any number of effaceables which are registered with the marginal and zonal or zonoid signatures. The results of effacement about the crowning edges are the data C G, and these are exactly given by our methods.

The marginal signatures of penesolids are of the form

$$[2a2b],\quad [\mathbf{2}a2b],\quad [\mathbf{3}a\mathbf{4}c],\quad [\mathbf{3}a\mathbf{a}\mathbf{A}c],$$
$$[2a\mathbf{A}b],\quad [\mathbf{A}a\mathbf{B}b],\ \&c.$$

The first has two marginal plane triangles and $a+b$ submargins; the second has one marginal plane triangle; the third has none. The fourth has $a+c$ *plane submargins* and $\mathfrak{a}$ *solid submargins.* We see that polyedra are removeable by the external primary effaceables in the third, which have a $(3+1)$-gonal and a $(4+1)$-gonal face; and that the fourth has been constructed by charging the plane marginal triangles of a subject penesolid, one with a polyedron having a $3+1$-gonal face, and the other either with a polyedron having an $(\mathfrak{A}+1)$-gonal face, or with a full penesolid having the solid margin $\mathfrak{A}$. The edge that crowns the first is the intersection of an $(a+3)$-gon and a $(b+3)$-gon. The edge that crowns the third may be the intersection of an $(a+3)$-gon and a $(c+6)$-gon, or of an $(a+4)$-gon and a $(c+5)$-gon, &c.

The polar summits being known for P-edra Q-acra and Q-edra P-acra, their reciprocal polar faces are of course known also.

This may suffice as an account of the data (C, D, G), except that we shall return to the construction of the plane and of the full reticulations.

We consider next the data B.

Analysis of the polar janal summits p$'$p$_{\prime}$ *of a* P-*edron* Q-*acron* W.

Let p' and $p_{\prime}$ be π-aces terminating a janal axis. The ordinary effaceables of either summit, which are what we have already considered about a polar p-ace, can be restored in one way only. The janal symmetry remains, though modified by such restoration.

There are in faces about p' a certain number of triaces collateral with $p_{\prime}$. These are $(\varpi \gtreqless 0)$ *rhombotomous triaces.* Let each be made collateral with p', and let the ϖ similar triaces in faces about $p_{\prime}$ be made collateral with p'. They are now all *rhombotomous tessaraces,* such that quadrilaterals $p'\rho p_{\prime}\rho_{\prime}$ are drawn through pairs $\rho\,\rho_{\prime}$ of the tessaraces. We have now a pair of janal π'-aces perfect both in their ordinary and their *peripolar effaceables,* which is registered in our table of *perfect janal poles.*

A *peripolar effaceable* is any ray drawn from a principal janal pole to a rhombotomous tessarace, which is always collateral with either pole, and may lose either of its effaceables.

If the two polar summits be entirely removed, there is laid bare either a janal polyedron or a janal reticulation at either pole. The two opposite reticulations have the same marginal signature, which differs in nothing that needs here be noticed from that of a polar reticulation. This is reducible at either pole by the external primary effaceables, or if it be a janal full reticulation, by its external secondary effaceables, either to a janal polyedron, or to a janal full reticulation, or to a *primitive*, or to a *fundamental janal reticulation*; and this reduction is possible in one way only.

A *fundamental janal reticulation* is made thus:—Let a *primary polar plane reticulation*, i. e. one which has no diagonals but the bases of its marginal triangles, and no summits besides those of those triangles, be laid on a mirror, with the triangles a little raised from the mirror. The reticulation and its image form a *fundamental janal reticulation*.

If the primary be turned in any way about an axis perpendicular to the mirror, while the image remains stationary, we have a *primitive janal reticulation*. The edges in the mirror, common to both polar faces, are the *submargins* of the polar faces, on which primitive submargins rhombotomous tessaraces can be planted at pleasure in janal coronation.

The marginal triangles of the fundamental are all doubled, and the bases of the pairs are zonal effaceables. Some only of the marginal triangles of the primitive are doubled, and the bases of the pairs are zonoid effaceables.

There is only one way in which a given janal reticulation can be constructed on a fundamental or on a primitive. The rules for construction differ little from the rules for polar reticulations, and we conceive that the marginal charges are imposed alike in the opposite polar faces, thus preserving janal or contrajanal symmetry. Inspection of our signatures gives us the result in all cases.

The janal reticulation is registered with its marginal signature, which of course is the same at either pole, with its zonal signatures, principal and secondary, and with its zonoid signature if it have secondary zoneless poles.

An account is kept, in all constructions on a fundamental or primitive, of *primitive plane submargins*, because it is on these only that rhombotomous tessaraces can be deposited in janal coronation. These

are the submarginal edges (not in triangles) of the fundamental or primitive.

The symmetry of a janal subject is modified in various ways by marginal charges, by coronation, or by effacements ordinary and peripolar.

The modifications are the following:—

On the $\frac{ri}{2}$-zoned heterozone janal subject we construct results,—

1. $\frac{r}{2}$-zoned heterozone,
2. $\frac{r}{2}$-zoned homozone,
3. r-ple zoneless monarchaxine janal,
4. r-ple monaxine monozone,
5. r-ple monaxine contrajanal.

On the $\frac{ri}{2}$-zoned homozone subject we construct results,—

1. $\frac{r}{2}$-zoned homozone,
2. r-ple zoneless monarchaxine janal,
3. r-ple monaxine contrajanal.

On the ri-ple zoneless monarchaxine janal subject we construct only r-ple monarchaxine janal results.

On the ri-ple monaxine monozone subject we construct results,—

1. r-ple monaxine monozone,
2. r-ple monaxine contrajanal.

On the ri-ple monaxine contrajanal subject we construct only r-ple monaxine contrajanal results.

In all these cases the constructions are given exactly by inspection of our signatures for every divisor r of ri: and we can both complete the table of perfect janal poles of $(P+e)$-edra Q-acra which we require, and register the janal results of e ordinary effacements in every possible way upon each of them, for every kind of symmetry.

Having thus registered the results of *ordinary effacement* about janal axes in a table of janal poles perfect only in peripolar effaceables, we have next to register the results of *peripolar effacement.*

The effect of a peripolar effacement is, that a rhombotomous tessarace becomes a rhombotomous triace, and that the secondary zone,

if there is one, is destroyed, a different janal symmetry being introduced.

Our signatures give us an exact account of all rhombotomous tessaraces introduced in coronation, and we can readily enumerate the results of peripolar effacement by mere inspection of those signatures.

All fundamental and primitive reticulations are given by general formulæ, as are also the primary plane reticulations on which they are founded, in terms of their marginal signatures, and of their zonal and zonoid signatures. Every kind of janal symmetry may be seen in these fundamentals and primitives.

Thus, on the supposition, as before, that we have tables of inferior polyedra, and of the plane and full reticulations, symmetrical or not, we can obtain completely the data B for P-edra Q-acra, and for Q-edra P-acra.

The data E are what the $\frac{r}{2}$-zoned homozone polyedra become when $r=2$. They are given by the theory of construction of the homozone poles for the general value of r; and each summit is registered with its zonal and zonoid signature and with its effaceables; and the results of effacement, ordinary or peripolar, are accurately known.

The data F are what the r-ple monaxine contrajanals become for $r=1$. We never descend so low as $r=1$, except in the construction of penesolids, viz. contrajanal anaxine penesolids. The janal anaxine pairs which crown these and other penesolids are data F. They are registered as perfect edges with their effaceables, and the janal results of effacement are also data F.

The data A are easily obtained. Every polyarchaxine reduces in one way only to a regular polyedron, on the principal faces of which it is constructed, always by its principal poles.

The effaced effaceables of a polyarchipolar summit (which exists either on a polyarchaxine or on its reciprocal) are all ordinary, and can be restored in one way only. We conceive them restored about all like archipoles. These poles being removed, a polyedron or a reticulation is laid bare, which has a marginal signature differing in nothing that needs here be noticed from that of a polar reticulation. By inspection of this signature, we can construct on it a given number of polyarchaxine reticulations of given signatures, and the process differs in nothing from the construction of a polar reticulation, except

that we conceive it effected in all the principal faces of the subject. We can crown, and register all summits with zonal or zonoid and marginal signatures and with effaceables, so that the results of ($e \overline{\gtrless} 0$) symmetrical effacements about all the principal axes are readily enumerated. These results are the data A.

It remains only that we return to the construction of the full and plane reticulations, from which all our marginal charges, if they be not polyedra, are selected.

The full reticulations are reducible always either to a nucleus line or to a nucleus polyedron, by sections in their external effaceables. The marginal signature shows always the edges removeable by such sections, and construction proceeds by the rule that no external effaceable of the subject shall be external in the result. All effaceables are here secondary.

The modifications of symmetry of polar monozone and asymmetric subjects and constructions are expressed in general formulæ, and the results are always registered with all signatures without ambiguity or repetition.

We never construct janal full reticulations except what have, as the word janal implies, a symmetry: and such constructions are always polar, except the objanal monozones and the contrajanal anaxine full penesolids. The only difference between these full constructions and that of mixed reticulations is, that no marginal triangles are handled or lost in the process; and that in the building of an r-ple repetition on a subject of ri-ple repetition, we descend to the value $r=1$, which gives the asymmetric full reticulations by the formulæ for the general value of r.

The plane reticulations have lastly to be considered.

All plane r-gonal penesolids ($r0f$) having $f-1(\overline{\gtrless} 2)$ diagonals, symmetrical or not, are given by general formulæ in terms of their marginal signature,

$$[2a2b], \qquad (r=4+a+b),$$

that is, in terms of ($rfab$). The line which crowns this signature is the intersection of a $(3+a)$-gon and a $(3+b)$-gon.

All polar plane reticulations and all monozones which have less than three epizonal edges in the zone, reduce in one way only to a polar or monozone primary, which has no diagonals except the bases

of its marginal triangles. On this primary as subject the reticulations in question are constructed by one operation, by charging its marginal triangles with marginal triangles of inferior reticulations, whereby two marginal triangles disappear for every charge imposed. The number and the symmetry of the constructions are always given by inspection of our signatures. The monozones which have more than two epizonal edges in the zone, reduce by section in the central epizonal or epizonals to two or to three inferior zoned reticulations.

We construct, conversely, a given marginal and zonal signature thus in every possible way, the number of constructions being always given by inspection of our tables. All polar reticulations having this zone and marginal signature will be formed by the process. The polar being already registered, the monozones are obtained by subtraction.

Perhaps the greatest difficulty in the theory of the polyedra is the enumeration of the M asymmetric plane reticulations which have the marginal signature

$$S=[2^{T}w],$$

where 2^{T} means simply 2T; and where T is the number of the marginal triangles, and w that of the submarginal edges, of which no two are contiguous. The reticulations to be found will be registered

$$R0F\ [2^{T}w]=M,$$

where

$$F-1-T=d$$

is the number of diagonals which are not bases of marginal triangles, and

$$R=2T+s+u, \quad =2T+w$$

is the number of the summits of the reticulation, of which s only are not summits in marginal triangles. If we erase the d diagonals, and also the s diaces that they may leave (summits of two edges), we have a primary reticulation,

$$R'0f\,[2^{T}u], \quad (R'=2T+u),$$

which may or may not be polar or monozone. If now on the u submargins of $R'0f$ we deposit s points (diaces) in every possible way, and then draw in the

$$(R-T=)(T+w=)(T+u+s)\text{-gon}$$

enclosed by the T marginal triangles d diagonals in every possible

way, none crossing another, so that one at least shall pass through each of the s points, we shall construct the asymmetric R0F in question among the results a certain number of times, namely, in every position of R0F in which erasure of the d diagonals and of the s diaces will reproduce the primary R$'0f$; and if R$'0f$ be polar, there may be many such positions of R0F for the same position of R$'0f$. And it is evident that we shall construct equally every plane reticulation R0F of every symmetry which can reduce by the same process of effacement to the same primary R$'0f$.

Nothing is easier than to determine the number of asymmetric constructions thus obtained of R0F $[2^{\mathrm{T}}w]$, if the number of all possible ways of drawing the d diagonals can be found.

We have to employ in turn every possible partition of the s points, of which one is

$$s = a_1 + a_2 + a_3 + \ldots + a_m \; (m \overline{\overline{<}} u),$$
$$(a_c \overline{\overline{<}} a_c + 1), \quad (a_1 > 0).$$

There is a given number of ways of depositing a_1 points on any one of the u submargins, a_2 points on any other, &c.

All that is difficult is to determine, when a disposition of the $a_1 + a_2 + \ldots + a_m$ points is made, in how many ways d diagonals can be drawn in the

$$(\mathrm{T} + w =) r\text{-gon},$$

so that one at least shall pass through each of the s points. Let this number be

$$rd_{a_1 a_2 a_3 \ldots}.$$

It is given always by the equations following:—Let $a_1 = 1$; then

$$rd_{a_1 a_2 a_3 \ldots} = rd_{a_2 a_3 \ldots} - (r-1) d_{a_2 a_3 \ldots} - (r-1)(d-1)_{a_2 a_3 \ldots}.$$

Let $a_1 > 1$; then

$$rd_{a_1 a_2 a_3 \ldots} = rd_{(a_1 - 1) a_2 a_3 \ldots} - (r-1)(d-1)_{(a_1 - 2) a_2 a_3 \ldots} - (r-1) d_{(a_1 - 1) a_2 a_3 \ldots},$$

where

$$rd_0 = \frac{\Pi(r+d-1)\Pi(r-3)}{\Pi(r-1)\,\Pi(d+1)\,\Pi(r-d-3)\,\Pi(d)}$$

is what I have called in my memoir "On the k-partitions of the R-gon and the R-ace" (Phil. Trans. 1857), the $(d+1)$-divisions of the r-gon.

This number $rd_{a_1 a_2 a_3 \ldots}$ being thus given for every partition of s,

the asymmetric plane reticulations $R0F[2^T w]$ are given, whatever be the primaries $R'0f$, symmetric or asymmetric, to which they are reducible by the above process of effacement.

And with these we have a complete solution, perfectly easy to calculate and register, of the problem of the *classification and enumeration of the* P-*edra* Q-*acra*.

The memoir, which I have the honour to present to the Royal Society, contains all the general formulæ of this solution.

There is nothing to prevent our registering in *all* marginal signatures the exact order of the margins. If this be done, we can crown every reticulation registered by a closed polygon A, made by connecting margins so that no linear section shall remain. The faces collateral with A would be at the same time constructed, and could be registered; that is, we could register all the faces collateral with any face in their order. And in crowning by summits marginal signatures so registered, we could determine and register the faces of every p-ace of the P-edra Q-acra in their order.

The methods above given are applicable to this more laborious registration of results. If this more tedious process be adopted, the construction of the P-edra Q-acra will be an easy matter.

I am not aware that anything has been printed on the subject of this theory beyond what I have attempted in the 'Philosophical Transactions,' and in the 'Memoirs of the Literary and Philosophical Society of Manchester,' except a short attempt made some three years ago by M. Poinsot to sketch a beginning of the investigation, which appeared in the 'Comptes Rendus.' The attempt was well made, but the results given were not quite accurate. I have it not at hand; but I know that there is a defective enumeration of the simple solids there considered.

I have enumerated the polyedra of not more than 18 edges by this method, and I hope shortly to publish the classification and enumeration of the polyedra of 20 edges and under.